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TRANSFER OF HEAT OR MASS TO PARTICLES IN FIXED AND FLUIDISED BEDS

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Abstract—Experimental measurements of heat transfer to particles in fixed beds show either a constant value of the Nusselt group as the Reynolds number is reduced or, if axial dispersion has been neglected, the Nusselt group decreases to zero. A quantitative analysis of particle to fluid heat transfer on the basis of a stochastic model of the fixed bed leads to a constant value of the Nusselt group at low Reynolds number. When the analytical equation is included as an asymptotic condition, an expression is derived that describes the dependence of Nusselt group upon Reynolds number. The expression is extended to describe mass and heat transfer to fixed and fluidised beds of particles within the porosity range of 0.35–1.0. Both gas and liquid phase transfer groups are correlated up to a Reynolds number of 10^5.

NOMENCLATURE

c, heat capacity for unit mass;
d, diameter of particle;
D, diffusion coefficient;
D_H, hydraulic diameter;
e, porosity of bed;
h, heat-transfer coefficient;
h_m, mass-transfer coefficient;
J_0, J_1, Bessel function of first kind, zero and first order;
k, constant defined by equation (1.8);
Nu, Nusselt group hd/λ;
q, heat flux;
q_1, rate of heat evolution in unit volume of fluid;
Re, Reynolds group $dU\rho/\mu$;
Sc, Schmidt group $\mu/(\rho D)$;
Sh, Sherwood group $h_m d/D$;
St, Stanton group $h/(c\rho U)$;
t, time;
T, temperature;
T_0, wall temperature;
u_0, axial velocity;
U, superficial velocity in fixed bed;
u, v, w, component velocities;
x, y, z, co-ordinates.

Greek symbols

α, root of equation (3.2);
η, co-ordinate;
λ, thermal conductivity;
ρ, density;
ψ, defined by equation (3.3);
μ, viscosity.

INTRODUCTION

ESTIMATES of the Nusselt group for heat transfer between fluid and particles in fixed beds at low Reynolds numbers have been reported in a number of recent publications [1–5]. The values of Nusselt group are shown in Fig. 1 as functions of Reynolds number; the fixed bed porosities were about 0.4. It is obvious at once that values reported by some investigators differ from others by several orders of magnitude at the same Reynolds number when that number is 10 or less. Evidently the experimental results may be divided into two distinct classes; in one the Nusselt group approaches a value in the range of 1–10 as the Reynolds number is reduced, while in the larger second class the Nusselt group approaches zero.

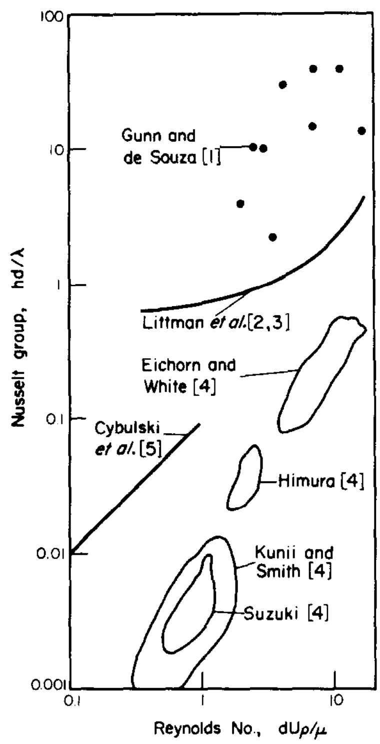

FIG. 1. The dependence of Nusselt group upon Reynolds number according to several investigators.

A variety of experimental methods were used by the several investigators including frequency response [1–3], steady state axial conduction [4] and steady radial and axial conduction [5], so that some differences might be expected because of the variety of experimental conditions. However, it is clear that the variations in Fig. 1 are more extreme than would be expected from this cause alone, and it is probable that differences in the interpretation of experimental measurements are more significant.

It is difficult to measure heat-transfer rates and temperature differences between the particle surface and the fluid in a fixed bed, and therefore values of the coefficients are often determined by comparing experimental measurements of temperature in the bed with theoretical distributions calculated from the differential equations describing transfer processes in fixed beds. The important factor that is mainly responsible for the variation in Fig. 1 is that experimental measurements have been interpreted according to different models of the transfer processes.

Four of the sets of investigators [2–5] interpreted their experiments in terms of a model that included separate dispersive fluxes in the solid and fluid phases together with interphase transfer; both solid and fluid phases were assumed to be continuous. However, in analysing their experiments Cybulski, van Dalen, Verkerk and van der Berg [5] neglected axial dispersion in the fluid phase and ignored a thermal resistance at the cylindrical heated surface of the fixed bed that has been found significant in similar experiments [6, 7].

The fluid phase is continuous in single phase flow, but in fixed beds of particles the solid phase is not continuous so that the introduction in a model of an independent dispersive flux in the solid phase is redundant and probably misleading. As thermal dispersion in the fixed bed takes place both in the fluid and in the solid phase, but the fluid phase alone is continuous, it is more logical to define the dispersion coefficient for the heterogeneous fluid–solid matrix. The dispersive flux is then taken to be proportional to the temperature gradient in the continuous phase since heat flow through the solid passes into the fluid phase at particle boundaries. This approach is the basis of the model adopted by Gunn and de Souza, and it is consistent with the study of diffusion in heterogeneous phases initiated by Maxwell for an electrical analogue of diffusion, and reviewed for example by Barrer [8]. In the limit at low flow rates, the thermal resistance of the medium is determined by the molecular conductivities and static structures of the phases and it is an important feature of a model that the limiting behaviour of the porous medium should be portrayed correctly. The model used by Gunn and de Souza has the correct limiting behaviour, and includes the effect of intraparticle conduction and fluid to particle transfer. The significant differences between this and other models are to be found in the treatment of dispersion in the fluid and solid.

The greatest differences are to be found between models that include thermal dispersion and those that do not, particularly when the Reynolds number is low. The effect of omitting thermal dispersion is to give low values of the Nusselt group when calculated from experiment, and this has been pointed out by several authors [1, 9, 10]. Kunii and Suzuki [4] applied a correction to their experimental measurements to account for the effect of axial dispersion, but Gunn and de Souza [1] have shown that the equation used both by Kunii and Suzuki, and Kunii and Smith [11] contains an important inconsistency, and the dependence of the Nusselt group upon the square of the Reynolds group in their equation is due to the neglect of axial dispersion.

The pattern of experimental results shown in Fig. 1 has been the stimulus for several theoretical studies of the interaction between convection and heat transfer [4, 10, 12–14]. Kunii and Suzuki suggested that fluid channelling through the packed bed might be important, and they found a linear dependence of Nusselt number upon the Reynolds group for different degrees of channelling; the influence of axial thermal dispersion was not considered. They found it necessary to postulate extensive channelling at low Reynolds numbers to bring experiment and theory into agreement, although significant channelling in fixed beds is not likely unless fluidisation of solids takes place, and fluidisation did not apparently occur in their experiments. Nelson and Galloway [13] considered a mechanism of transient conduction involving concepts from boundary-layer and penetration theory, and the boundary layer also formed part of the analysis of Pfeffer and Happel [14]. It should be stated that conditions necessary for the boundary-layer equations to be valid are not satisfied in fixed beds of particles at a porosity of 0.4.

There are also available in the literature expressions such as that of Ranz [15] who postulated that the limiting value of Nusselt group for particles in fixed beds should be 2, the same value as that found for the

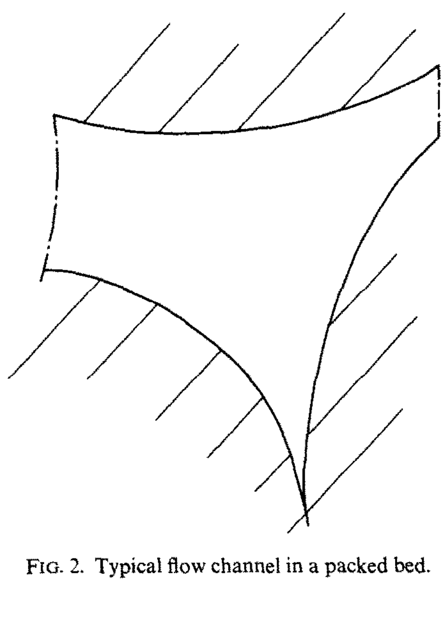

FIG. 2. Typical flow channel in a packed bed.

isolated particle at low Reynolds numbers. This postulate has not been supported either by analysis or by experiment; however the existence of an asymptotic condition of constant Sherwood group at low Reynolds number has been suggested for the analogous situation in mass transfer [16, 17].

The purpose of this paper is to consider the interaction of convection, thermal conduction and interphase transfer in fixed beds on the foundations of some statistical properties of the flow field, and of the diffusion field. By means of the statistical representation, analytical results derived for well-defined conditions of flow and diffusion are applied to the description of convective and diffusive transfer in random-packed beds. It is found that the experimental results for fixed beds, fluidised beds, and single particles may be described by one equation that contains both analytical and experimental conditions, and is valid for both heat and mass transfer.

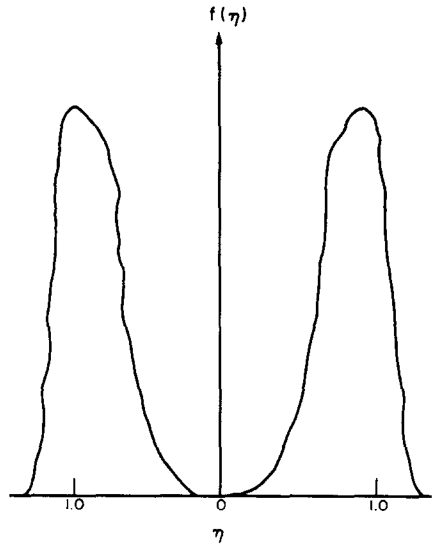

FIG. 3. Probability density of the distribution of surface area.

STATISTICAL PROPERTIES OF THE FLOW AND HEAT DIFFUSION FIELDS

The fixed bed filled with particles may be considered as an assemblage of flow channels through which fluid flows at velocities that are greatest in the central part of the channel and smallest near the bounding surfaces. The channels are bounded either by the surfaces of particles, or by the common surfaces of neighbouring flow channels; a typical channel is shown in Fig. 2. The distance across a channel will be about equal to the diameter of a particle at the widest part, and about half a diameter at the narrowest part, when the bed is not consolidated. Two orthogonal co-ordinates are chosen in a plane normal to the axial direction so that each divides the cross-section of a flow channel into two equal parts, and the intersection is the central point of the flow channel.

As the orientations of surfaces in the bed are random, the probability of one of the co-ordinates intersecting a surface at a particular distance from the central point is independent of the direction of the co-ordinate in the normal plane; the position of the plane is defined by the axial co-ordinate x. The probability of intersection for a particular x plane is therefore a function of the length η of the vector from the central point, and the distribution about the origin will correspond to the distribution of surface area as a function of η. The probability density of the distribution may be obtained from the assemblage of flow channels.

It is convenient to consider the flow and heat diffusion properties of the assemblage in two parts. In the first part some averaged properties of the distribution are considered and the consequences for heat flow are derived in the succeeding sections of the paper. Such a study will give the broad dependence of heat transfer within the bed upon the important physical parameters, but will not include detailed effects that are characteristic of the geometries of the flow and diffusion fields. The detailed effects are simulated by a stochastic model of the packed bed that preserves the characteristic geometry.

Consider the probability density of the distribution of surface area obtained from the assemblage of flow channels. If an average is taken over all x, the density $f(\eta)$ will be independent of x, and $f(\eta)\,d\eta$ will be the proportion of surface area in the average flow channel that lies within a distance of $\eta - \eta + d\eta$ from the central point. The distribution is symmetric and it is illustrated in Fig. 3.

Because the distribution has been obtained by averaging over the random variable x, radial flow velocities in the channel are averaged out, and therefore the radial components of velocity are taken to be zero everywhere within the bed. The distribution of axial velocity within the average flow channel is determined by the equation of motion for flow when the bounding surface is given by the distribution shown in Fig. 3. The mathematical form of the distribution is not known, but the distribution may be characterised by its moments, and both the zeroth and the first moments are known.

$$\mu_0 = \int_0^H f(\eta)\,d\eta = 1 \tag{1.1}$$

$$\mu_1 = \int_0^H \eta f(\eta)\,d\eta = \bar{\eta} \tag{1.2}$$

where H is the upper limit of η for the distribution.

Equation (1.1) states that all of the surface area within the bed is contained in the distribution. Equation (1.2), the first moment, gives the average value for η for the distribution and it may be determined from the geometrical properties of the particle: Let the number of flow channels within the unit cross section be n, then the bed voidage e is related to n and $\bar{\eta}$ by,

$$n\pi\bar{\eta}^2 = e. \tag{1.3}$$

The surface area of the particles in unit volume $6(1-e)/d$ is related to n and $\bar{\eta}$ by

$$n2\pi\bar{\eta} = 6(1-e)/d \tag{1.4}$$

so that

$$\bar{\eta} = \frac{de}{3(1-e)}. \tag{1.5}$$

In this simple stochastic representation, the surface distribution of Fig. 3 is assumed concentrated at $\eta = \bar{\eta}$. The distribution of axial velocity in the fixed bed described by the model is a maximum at $\eta = 0$ falling parabolically with η to zero at $\eta = \bar{\eta}$. The radial component of velocity is taken to be zero everywhere within the bed.

The analysis of heat transfer within the bed based upon this approximation is presented in the next two sections of the paper. It is clear that the heat-transfer characteristics of the packed bed will differ from those found for the approximation because the disposition of heat-transfer surfaces in relation to the flow field is significantly different in the packed bed. In particular the true Nusselt groups will differ.

The geometrical characteristics of the packed bed may be examined by cutting sections of a flow channel in planes normal to the axis of flow at random values of the axial co-ordinate x. Figure 4 shows three sections that have been chosen by this procedure when the sections have been taken through an arrangement of spheres. For each section the ratio of area available to flow to the total channel area will lie between a lower bound and the upper bound of 1.

Consider the situation in which fluid moves slowly through the cross section generating heat at a constant rate in unit volume and rejecting heat at the solid surfaces that are held at zero temperature. The differential equation that describes the temperature distribution in the section is

$$\lambda\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+q_1 - \rho c\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right) = 0 \tag{1.6}$$

where q_1 is the constant rate of heat evolution in unit volume. As the fluid velocity in the section is reduced the final three terms in equation (1.6) become of small importance, and at the same time the relative magnitude of $\partial^2 T/\partial x^2$ declines so that equation (1.6) takes the simpler form:

$$\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_1}{\lambda} = 0. \tag{1.7}$$

The fluid-mechanical analogue of this equation has been studied by Gunn and Darling [18] for the three sections shown in Fig. 4. For the quantities shown in equation (1.7) their solution takes the form,

$$k = \frac{D_H^2}{2\lambda\bar{T}}(q_1) \tag{1.8}$$

where $\bar{T}$ is the average temperature in the section, k is a dimensionless constant characteristic of the section, and D_H is four times the ratio of the fluid cross sectional area to the surface area.

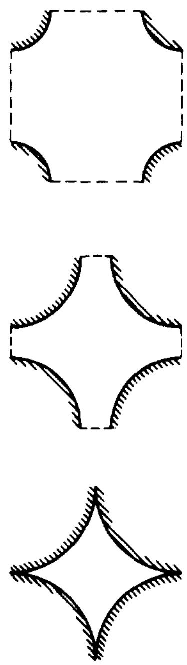

FIG. 4. Sections obtained by random cutting in the plane normal to flow.

Table 1 shows the salient results of their calculations.

Table 1. Heat-transfer characteristics of the sections shown in Fig. 4 [18]

Section	D_H	k	kd^2/D_H^2	e_1 volume fraction of fluid in section
1	0.273d	6.5	87.2	0.215
2	0.766d	14.5	24.7	0.490
3	0.829d	23.0	33.5	0.65

The dependence of kd^2/D_H^2 upon the planar voidage e_1 may be expressed as a continuous quadratic function that satisfies the three points given in Table 1. The function is,

$$\frac{kd^2}{D_H^2} = 204.2 - 683.86e_1 + 648.07e_1^2. \tag{1.9}$$

The minimum planar voidage of the sections is 0.215, and the porosity of a bed packed with spheres is about 0.37; if the distribution of sections within the packed bed is uniform between the upper and lower limits of planar voidage U and 0.215, then the upper limit of planar voidage is defined so that $0.5(0.215+U) = 0.37$, i.e. U is 0.525. Hence the mean value of kd^2/D_H^2 is given by the expression

$$\left(\overline{\frac{kd^2}{D_H^2}}\right) = \frac{1}{(0.525-0.215)}\int_{0.215}^{0.525}\frac{kd^2}{D_H^2}\,de_1. \tag{1.10}$$

On substitution of equation (1.9) it is found that,

$$\left(\overline{\frac{kd^2}{D_H^2}}\right) = 45.08. \tag{1.11}$$

The rate of heat evolution in unit volume of the packed bed is $q_1 e$, the surface area available for heat transfer between fluid and particles is $6(1-e)/d$, so that in terms of quantities defined in equation (1.8), the Nusselt group is

$$\frac{hd}{\lambda} = \left(\frac{q_1 ed}{6(1-e)\bar{T}}\right)\frac{d}{\lambda}. \quad (1.12)$$

However for the circular section D_H is given by

$$D_H = \frac{2ed}{3(1-e)} \quad (1.13)$$

so that the corresponding value of kd^2/D_H^2 is

$$\frac{kd^2}{D_H^2} = \frac{16d^2}{\{4ed/[6(1-e)]\}^2} = 36\left(\frac{1-e}{e}\right)^2 = 104.4 \quad (1.14)$$

as the value of k for the circular section is 16 [18]. Taking into account equations (1.11), (1.13) and (1.8), the ratio ϕ of the Nusselt group for heat transfer in the packed bed to the Nusselt group for heat transfer to fluid flowing in circular tubes of hydraulic diameter equal to that of the fixed bed is:

$$\phi = \frac{45.08}{104.4} = 0.43. \quad (1.15)$$

If the heat-transfer characteristics of tubular sections at low flow rates are now determined, the Nusselt group for the packed bed is then found as the product of ϕ and the Nusselt group for the circular section. The complementary analysis for heat transfer within the bed based upon this approximation is presented within the next two sections of the paper.

THE INTERACTION BETWEEN CONVECTION, CONDUCTION AND HEAT TRANSFER BETWEEN FLUID AND PARTICLES

The transport of heat within a flow section is determined by the axial and radial thermal conductivities, and by the velocity distribution that is parabolic about the main axis of the flow field for the equivalent section. When conditions within a flow section are steady with time, the partial differential equation that describes the transport of heat is,

$$\lambda\left(\frac{\partial^2 T}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial T}{\partial \eta} + \frac{\partial^2 T}{\partial x^2}\right) - 2\rho c u_0\left[1-\left(\frac{\eta}{\bar{\eta}}\right)^2\right]\frac{\partial T}{\partial x} = 0. \quad (2.1)$$

Heat transfer takes place from fluid to particles at the boundary of the region because the fluid temperature approaches that of the surface of the solid at the boundary. The boundary condition to equation (2.1) is

$$T = T_0(x) \quad \text{at} \quad \eta = \bar{\eta}. \quad (2.2)$$

Once the temperature distribution has been found the particle to fluid heat-transfer coefficient h may be determined from the definition:

$$h = \frac{q}{T_0 - \bar{T}} \quad (2.3)$$

where $\bar{T}$ is the average fluid temperature, sometimes known as the "mixing cup" temperature.

The temperature of the particles $T_0(x)$ will, in general, be determined by subsidiary conditions that define, for example, heat evolved in a chemical reaction. However, as our purpose is to investigate the dependence of the heat-transfer coefficient upon the flow and thermal properties of the system outside the particles, it is sufficient to define $T_0(x)$ so that a temperature difference is created between fluid and solid that leads to heat transfer. A suitable function that satisfies this criterion is,

$$T = 0 \text{ at } \eta = \bar{\eta},\ x < 0$$
$$T = T_0, \text{ a constant at } \eta = \bar{\eta},\ 0 \leq x < \infty. \quad (2.4)$$

An exact analytical solution to equation (2.1) defined by boundary conditions (2.4) is not available. However, if the velocity in the bed is assumed constant with η rather than parabolic an exact solution is available that includes the effect of thermal conduction but not of velocity distribution, while if axial thermal conduction is neglected an exact analytical solution is available that describes the effect of velocity distribution. It will be shown that the dependences of the heat-transfer coefficient upon the thermal and flow properties of the system found from the analytical relations are consistent and agree at low Reynolds number.

THE INFLUENCE OF AXIAL THERMAL CONDUCTION AND VELOCITY DISTRIBUTION UPON THE HEAT-TRANSFER COEFFICIENT

The analytical solution to equation (2.1) when $2u_0[1-(\eta/\bar{\eta})^2]$ is replaced by u_0, and the boundary conditions are defined by equation (2.4) has been given by Carslaw and Jaeger [19] in series form. The temperature distribution for $x > 0$ is:

$$\frac{T}{T_0} = 1 - \sum_{s=1}^{\infty} \frac{1}{\bar{\eta}\alpha_s}\frac{J_0(\eta\alpha_s)}{J_1(\bar{\eta}\alpha_s)}\frac{[\psi + (\psi^2+\alpha_s^2)^{1/2}]}{(\psi^2+\alpha_s^2)^{1/2}}\exp\{x[\psi - (\psi^2+\alpha_s^2)^{1/2}]\} \quad (3.1)$$

where the α_s are the successive roots of

$$J_0(\bar{\eta}\alpha) = 0 \quad (3.2)$$

and

$$\psi = \rho c u_0/2\lambda. \quad (3.3)$$

From this equation the heat flux may be found:

$$q = -\lambda\left(\frac{\partial T}{\partial \eta}\right)_{\eta=\bar{\eta}} = T_0 \sum_{s=1}^{\infty} \frac{\lambda}{\bar{\eta}} \frac{[\psi + (\psi^2+\alpha_s^2)^{1/2}]}{(\psi^2+\alpha_s^2)^{1/2}} \exp\{x[\psi - (\psi^2+\alpha_s^2)^{1/2}]\}. \tag{3.4}$$

The calculated temperature distribution equation (3.1) includes the effect of axial conduction but not of the velocity distribution of equation (2.1). If it is assumed that the temperature distribution that is the solution to equation (2.1) is approximated by equation (3.1), then the "mixing cup" temperature of the fluid is found from the integral,

$$\bar{T} = \frac{1}{\pi \bar{\eta}^2 u_0} \int_0^{\bar{\eta}} 2\pi\eta \cdot 2u_0 \left[1 - \left(\frac{\eta}{\bar{\eta}}\right)^2\right] T \, d\eta \tag{3.5}$$

where T is given by equation (3.1), so that

$$\bar{T} = \frac{4T_0}{\bar{\eta}^2} \int_0^{\bar{\eta}} \eta \left[1 - \left(\frac{\eta}{\bar{\eta}}\right)^2\right] \left\langle 1 - \sum_{s=1}^{\infty} \frac{1}{\bar{\eta}\alpha_s} \frac{J_0(\eta\alpha_s)}{J_1(\bar{\eta}\alpha_s)} \frac{[\psi + (\psi^2+\alpha_s^2)^{1/2}]}{(\psi^2+\alpha_s^2)^{1/2}} \exp\{x[\psi - (\psi^2+\alpha_s^2)^{1/2}]\} \right\rangle d\eta \tag{3.6}$$

and on evaluating the integral

$$\bar{T} = T_0 \left\langle 1 - \sum_{s=1}^{\infty} \frac{16}{\bar{\eta}^4 \alpha_s^4} \frac{[\psi + (\psi^2+\alpha_s^2)^{1/2}]}{(\psi^2+\alpha_s^2)^{1/2}} \exp\{x[\psi - (\psi^2+\alpha_s^2)^{1/2}]\} \right\rangle. \tag{3.7}$$

The heat-transfer coefficient may now be found by substituting equations (3.4) and (3.7) into (2.3). The result is:

$$h = \frac{\lambda}{\bar{\eta}} \frac{\displaystyle\sum_{s=1}^{\infty} \frac{[\psi + (\psi^2+\alpha_s^2)^{1/2}]}{(\psi^2+\alpha_s^2)^{1/2}} \exp\{x[\psi - (\psi^2+\alpha_s^2)^{1/2}]\}}{\displaystyle\sum_{s=1}^{\infty} \frac{16}{\bar{\eta}\alpha_s^4} \frac{[\psi + (\psi^2+\alpha_s^2)^{1/2}]}{(\psi^2+\alpha_s^2)^{1/2}} \exp\{x[\psi - (\psi^2+\alpha_s^2)^{1/2}]\}}. \tag{3.8}$$

It will be noticed that the numerator of equation (3.8) does not converge for $x = 0$; this is due to the discontinuity in the surface temperature at this plane. However, at low velocities when x is of the order of $\bar{\eta}$ and greater, both numerator and denominator converge rapidly so that both are dominated by the first terms of the series. On substituting for $\bar{\eta}$ from equation (1.5) when x is of the order of $\bar{\eta}$ or greater;

$$\frac{hd}{\lambda} = \frac{3(1-e)}{16e} (\bar{\eta}\alpha_1)^4 \tag{3.9}$$

and on introducing the value of the first root of equation (3.2),

$$\frac{hd}{\lambda} = \frac{6.27(1-e)}{e}. \tag{3.10}$$

As equation (1.5) shows that $\bar{\eta}$ is about $0.2d$, the value of the Nusselt group within the packed bed is substantially constant when the velocity is low, and the value shown by equation (3.10) represents an asymptote that is approached as the velocity is reduced.

The asymptotic value is not affected by the inclusion of axial conduction in the analysis. When the above analysis is repeated for the condition in which fluid flows at constant velocity, but without the effect of axial conduction, the temperature distribution in the fluid is,

$$\frac{T}{T_0} = 1 - \sum_{s=1}^{\infty} \frac{2}{\bar{\eta}\alpha_s} \frac{J_0(\eta\alpha_s)}{J_1(\bar{\eta}\alpha_s)} \exp\left(-\frac{\alpha_s^2 x}{2\psi}\right) \tag{3.11}$$

where the α_s are the successive roots of equation (3.2). From this equation the heat flux may be found

$$q = -\lambda\left(\frac{\partial T}{\partial \eta}\right)_{\eta=\bar{\eta}} = T_0 \sum_{s=1}^{\infty} \frac{2\lambda}{\bar{\eta}} \exp\left(-\frac{\alpha_s^2 x}{2\psi}\right) \tag{3.12}$$

and when $\bar{T}$ is defined by equation (3.5), on substituting for T from equation (3.11)

$$\bar{T} = \frac{4T_0}{\bar{\eta}} \int_0^{\bar{\eta}} \eta \left[1 - \left(\frac{\eta}{\bar{\eta}}\right)^2\right] \left[1 - \sum_{s=1}^{\infty} \frac{2}{\bar{\eta}\alpha_s} \frac{J_0(\eta\alpha_s)}{J_1(\bar{\eta}\alpha_s)} \exp\left(\frac{-\alpha_s^2 x}{2\psi}\right)\right] d\eta \tag{3.13}$$

so that when the integral is evaluated

$$\bar{T} = T_0 \left[1 - \sum_{s=1}^{\infty} \frac{32}{\bar{\eta}^4 \alpha_s^4} \exp\left(\frac{-\alpha_s^2 x}{2\psi}\right)\right] \tag{3.14}$$

to give the following expression for the heat-transfer coefficient;

$$h = \frac{\lambda}{\bar{\eta}} \frac{\sum_{s=1}^{\infty} \exp\left(\frac{-\alpha_s^2 x}{2\psi}\right)}{\sum_{s=1}^{\infty} \frac{16}{\bar{\eta}^4 \alpha_s^4} \exp\left(\frac{-\alpha_s^2 x}{2\psi}\right)}. \tag{3.15}$$

It is clear that equation (3.15) gives the same asymptotic expression as equation (3.8) at low Reynolds number. The numerator does not converge at $x = 0$ also because of the discontinuity in temperature at that plane.

The solution to equation (2.1) when axial conduction is not considered, but the full form of the equation is otherwise retained, has also been given by Graetz [20, 21]. When expressed in terms of the "mixing cup" temperature of equation (3.5);

$$\frac{\bar{T}}{T_0} = 1 - 8\left[0.10238 \exp\left(-14.6272 \frac{x}{8\bar{\eta}^2\psi}\right) + 0.0122 \exp\left(-89.22 \frac{x}{8\bar{\eta}^2\psi}\right)\ldots\right]. \tag{3.16}$$

The heat-transfer coefficient may be found from this equation,

$$h = \frac{q}{T_0 - \bar{T}} = \frac{\pi\bar{\eta}^2 u_0 \rho c}{T_0 - \bar{T}}\left(\frac{d\bar{T}}{dx}\right)\delta x/(2\pi\bar{\eta}\delta x) = \frac{\bar{\eta} u_0 \rho c}{2(T_0 - \bar{T})}\frac{d\bar{T}}{dx}. \tag{3.17}$$

On substitution for $\bar{\eta}$ from equation (1.5), and for $\bar{T}$ from equation (3.16) into equation (3.17) the Nusselt group is found to be

$$\frac{hd}{\lambda} = \frac{3(1-e)}{2e} \frac{\left\{0.374 \exp\left[-14.6272 \frac{x}{8\bar{\eta}^2\psi}\right] + 0.272 \exp\left[-89.22 \frac{x}{8\bar{\eta}^2\psi}\right]\ldots\right\}}{\left\{0.10238 \exp\left[-14.6272 \frac{x}{8\bar{\eta}^2\psi}\right] + 0.0122 \exp\left[-89.22 \frac{x}{8\bar{\eta}^2\psi}\right]\ldots\right\}}. \tag{3.18}$$

At low velocities the series in numerator and denominator converge rapidly, and for this condition the low Reynolds number asymptote of the Nusselt group is,

$$\frac{hd}{\lambda} = 5.48 \frac{(1-e)}{e}. \tag{3.19}$$

Since equation (3.8) and equation (3.15) have the same low Reynolds number asymptotes it might be expected that the asymptote of the complete solution to equation (2.5) would be well represented by equation (3.19) because the inclusion of axial conduction in the analysis does not change the asymptote of the Nusselt group at low Reynolds number. In any event the asymptotic value, equation (3.10), is similar to that of equation (3.19) and therefore there can be some confidence not only in the form of the relationship as an asymptotic solution to equation (2.1), but also in the limiting value of the Nusselt group.

A WIDE-RANGING CORRELATION FOR THE NUSSELT GROUP IN FLUID AND PARTICLE SYSTEMS

The limiting value of the Nusselt group at low Reynolds number in fixed beds may be found by modifying equation (3.19) to introduce the particular geometry of the surfaces as given by equation (1.15). The limiting value is,

$$\frac{hd}{\lambda} = 2.36 \frac{(1-e)}{e}. \tag{4.1}$$

This equation is one of four asymptotic relations that delineate the bounds of the Nusselt group for heat transfer to particles at low and high Reynolds number, and low and high porosities. The other three limiting relations are the value of Nusselt group for the single particle at low Reynolds number, and the experimentally-established dependence of the Nusselt group for single particles and fixed beds upon Reynolds and Prandtl groups at high Reynolds number. Equation (4.2) is based upon all four relations and some evidence for the dependence of the Nusselt group upon porosity.

$$Nu = (7 - 10e + 5e^2)(1 + 0.7Re^{0.2}Pr^{1/3}) + (1.33 - 2.4e + 1.2e^2)Re^{0.7}Pr^{1/3}. \tag{4.2}$$

The form of equation (4.2) has been chosen to satisfy not only the four limiting relationships, but also the condition,

$$\left(\frac{\partial Nu}{\partial e}\right) = 0 \text{ as } e \to 1 \tag{4.3}$$

a natural condition for particle systems in a high degree of expansion that also ensures that the Nusselt group is minimum at $e = 1$. The dependence of Nusselt group upon porosity suggested by equation (4.1) is not valid as the porosity of the bed approaches 1, and has therefore bee⁻ replaced.

A more thorough comparison of equation (4.2) with experiment can be made if experimental results for mass transfer are also considered. The basis for the inclusion of experimental results for mass transfer may be shown by considering the partial differential equation for the transfer of heat within a moving fluid. It is

$$\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) - \rho c\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}+\frac{\partial T}{\partial t}\right)=0. \quad (4.4)$$

The boundary condition is given by the temperature of the solid at the boundary of the fluid region, and the rate of heat transfer into the fluid at the boundary Q is

$$Q=\int\left(-\lambda\frac{\partial T}{\partial n}\right)\mathrm{d}A \quad (4.5)$$

where the integral is taken over the boundary of which $\mathrm{d}A$ is an element of area, and n is the normal at the boundary.

If the thermal diffusivity $\lambda/\rho c$ and temperature T are replaced by the molecular diffusion coefficient and the concentration, and the boundary condition of temperature is replaced by the same distribution of concentration, then the concentration distribution found as the solution to the partial differential equation, is identical to the temperature distribution found as the solution to the analogous problem. In particular, if the solution is used to calculate the Nusselt group as functions of Reynolds number and Prandtl group, the same dependence of Sherwood group upon the Reynolds number and Schmidt group will be found, since the fluxes of heat and mass, and mean temperatures and concentrations are analogous. The analogies will be inexact only if the physical properties vary, or if radiant heat transfer contributes.

Figure 5 shows the dependence of experimental measurements of the Nusselt group upon Reynolds number for air. At high Reynolds number the experimental results of Denton [22] are typical of heat-transfer measurements at high Reynolds number and equation (4.2) reduces to Denton's relationship

$$St = 0.72Re^{-0.30} \quad (4.6)$$

proposed for the Reynolds number range from 500 to 50 000. Denton's measurements were made for single spheres heated electrically, but the omissions of the effect of other particles and of the effect of axial dispersion are not as serious at high Reynolds number. Figure 5 also shows the experimental results of Turner and Otten [23], who included the effect of axial dispersion in their analysis, that confirm the form of equation (4.2) at high Reynolds number and also agree with the experimental results of Denton. At low Reynolds number, only those experimental results taken from analyses that have included the effect of axial dispersion have been considered; the experimental results of Littman and Sliva [3] have been included although they assumed that both the fluid and solid phases are continuous. At low Reynolds number, the scatter in the experimental points is due to the smaller sensitivity of the experimental response to the Nusselt group.

Figure 5 also includes some recent estimates of the Sherwood group at low Reynolds number reported by Miyauchi, Kataoka and Kikuchi [17] for a system in which the Schmidt group was 0.9.

Figure 6 shows the experimental results of Rowe, Claxton and Lewis [24] for heat transfer to a single isolated sphere in a flowing fluid. Values of Nusselt

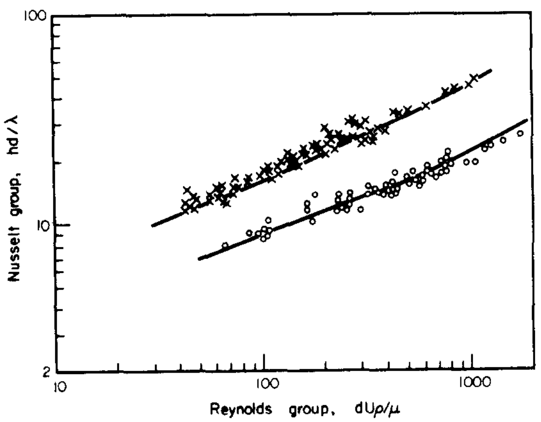

FIG. 6. Experimental results of Rowe, Claxton and Lewis for heat transfer to a single sphere compared with full lines calculated from equation (4.2). ×, water; ○, air.

group measured for heat transfer to water and heat transfer to air are shown on the same figure. Figure 7 shows that the experimental results of the same investigators for mass transfer from a single sphere to water and to air, correspond with equation (4.2). The good agreement evident in both figures shows that the exponent to the Schmidt or Prandtl group is correct at $\frac{1}{3}$, since a change to 0.4, for example, brings about a marked difference between the experimental data and the correlating equation.

Experimental investigations of particle–fluid heat transfer in liquid-fluidised beds are few in number, and

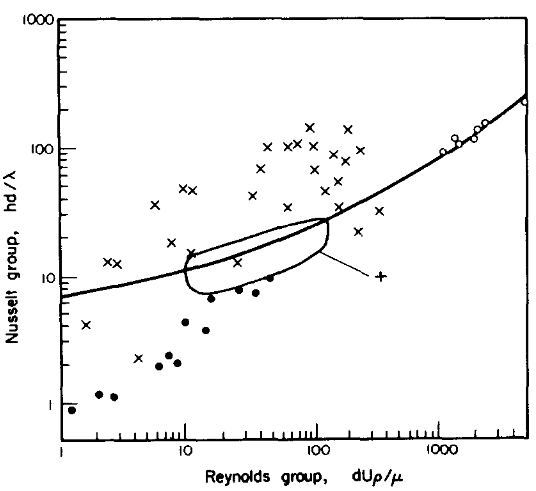

FIG. 5. Comparison of equation (4.2) with estimates of the Nusselt group and of the Sherwood group from experiments on fixed beds. Legend ×, Gunn and de Souza [1]; ○, Turner and Otten [23]; ●, Littman and Sliva [3]; +, Miyauchi *et al.* [17], (Sherwood group).

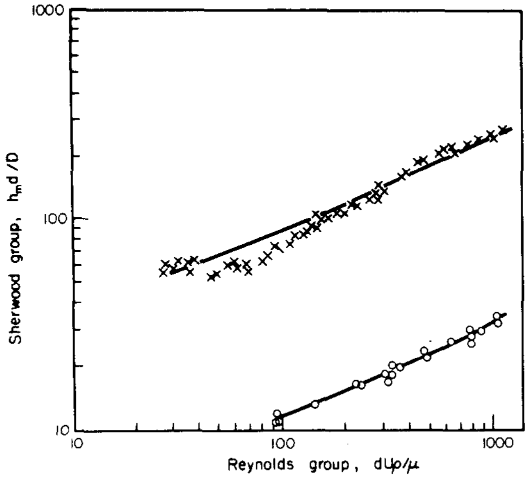

FIG. 7. Experimental results of Rowe, Claxton and Lewis [24] for mass transfer to a single sphere compared with equation (4.2). ×, water; ○, air.

similar experiments can show considerable differences. The experiments of Rowe and Claxton [25] included some mass-transfer measurements in liquid-fluidised beds, and although only a few active particles were used and dispersion was neglected, the values of the Sherwood group found were probably not seriously in error because the Reynolds number was high. The experimental results are shown in Fig. 8 plotted against porosity; the porosity was varied by changing the fluid velocity. The trend of the Sherwood group calculated from equation (4.2) is shown as the full line on the figure. The effect of increased porosity predicted by equation (4.2) is to reduce the value of the Sherwood group, but increased porosity in the fluidised bed is achieved by an increase in the particle Reynolds number, a factor that causes an increase in the value of the Sherwood group. Under the conditions of the investigation it appears that the two effects are to some extent counterbalancing, so that the value of the Sherwood group changes little over the range of porosity. Although widely scattered, the values of the experimental points in the figure are consistent with

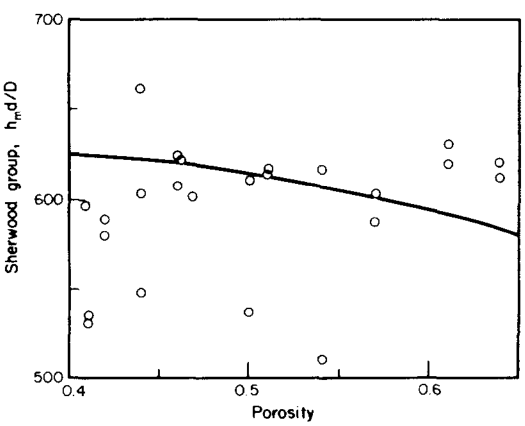

FIG. 8. Experimental results of Rowe and Claxton [25] for mass transfer in a liquid fluidised bed compared with equation (4.2).

little variation with porosity and their values are similar to those predicted by equation (4.2). The agreement to some extent confirms the form of the porosity function for the range of investigation and gives further justification to the mathematical form of the equation as the groups for heat and mass transfer to single particles, fixed beds and fluidised beds are described within the limits of experimental error.

The general form of equation (4.2) for heat and mass transfer is

$$N_T = (7 - 10e + 5e^2)(1 + 0.7Re^{0.2}M_T^{1/3}) + (1.33 - 2.4e + 1.2e^2)Re0.7M_T^{1/3} \quad (1.A)$$

where M_T is the Prandtl group when N_T is the Nusselt group, and M_T is the Schmidt group when N_T is the Sherwood group.

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TRANSFERT DE CHALEUR ET DE MASSE A DES PARTICULES DANS DES LITS FIXES OU FLUIDISES

Résumé—Des mesures expérimentales de transfert de chaleur à des particules dans des lits fixes montrent, lorsque le nombre de Reynolds est diminué, soit une valeur constante du groupe de Nusselt, soit une décroissance de celui-ci jusqu'à zéro si la dispersion axiale est négligée. Une analyse quantitative du transfert entre particule et fluide, sur la base d'un modèle stochastique de lit fixe, conduit à une valeur constante du groupe de Nusselt aux petits nombres de Reynolds. Lorsque l'équation analytique est introduite comme une condition asymptotique, on obtient une expression qui décrit la dépendance du groupe de Nusselt vis-à-vis du nombre de Reynolds. L'expression est élargie à la description du transfert de chaleur et de masse dans les lits fixes ou fluidisés de particules pour un domaine de porosité 0,35–1,0. Les groupes de transfert, aussi bien pour une phase liquide que gazeuse sont donnés pour un nombre de Reynolds atteignant 10^5.

WÄRME- ODER STOFFÜBERTRAGUNG AN PARTIKELN IM FEST UND WIRBELBETT

Zusammenfassung—Versuchsmessungen des Wärmeübergangs an Partikeln im Festbett zeigen entweder einen konstanten Wert für die *Nu*-Zahl, wenn die *Re*-Zahl verkleinert wird, oder die *Nu*-Zahl fällt auf Null ab, wenn die axiale Streuung vernachlässigt wird. Eine quantitative Analyse des Wärmeübergangs von Partikeln an das Fluid auf der Grundlage eines stochastischen Modells des Festbettes führt bei kleinen *Re*-Zahlen auf einen konstanten Wert der *Nu*-Zahl. Für den Fall, daß die analytische Gleichung als asymptotische Randbedingung mit einbezogen wird, wird ein Ausdruck abgeleitet, der die Abhängigkeit der *Nu*-Zahl von der *Re*-Zahl beschreibt. Dieser Ausdruck wird erweitert zur Beschreibung des Stoff- und Wärmeübergangs in Fest- und Wirbelbetten bei Partikeln in einem Durchlässigkeitsbereich von 0,35–1,0. Die Übergangszahlen für Gas- und Flüssig-Phase werden zueinander in Beziehung gesetzt bis zu Reynolds-Zahlen von 10^5.

ПЕРЕНОС ТЕПЛА ИЛИ МАССЫ К ЧАСТИЦАМ В НЕПОДВИЖНЫХ И ПСЕВДООЖИЖЕННЫХ СЛОЯХ

Аннотация — Экспериментальные измерения переноса тепла к частицам в неподвижном слое показывают, что критерий Нуссельта либо остается постоянным при уменьшении числа Рейнольдса, либо, если пренебречь аксиальной дисперсией, снижается до нуля. Количественный анализ переноса тепла от частицы к жидкости на основе стохастической модели неподвижного слоя дает постоянное значение числа Нуссельта при малых числах Рейнольдса. Использование аналитического уравнения в качестве асимптотического условия дает зависимость числа Нуссельта от числа Рейнольдса, которая применяется для описания тепло- и массопереноса к частицам в неподвижных и псевдоожиженных слоях с порозностью от 0,35 до 1,0. Корреляции переноса как газовой, так и жидкой фаз получены для значений числа Рейнольдса до 10^5.